



June 2020

Summer Math Work: AP Calculus AB

We hope you are enjoying your summer, but are also thinking about how to have a great school year next year.

Since AP Calculus AB is an Advanced Placement course, we would like to minimize the amount of class time spent reviewing old material so that we can get off to a running start with the powerful and beautiful ideas of the calculus.

The attached problem set is intended to address this need. It provides a quick reminder of the key Pre-Calculus topics needed for success. **All AP Calculus AB students are required to complete the problem set to be turned in to their teacher as a homework assignment when returning to school in the fall.** This is the “summer reading” for your mathematics course. Be sure to show your work clearly in completing the problems. One of the keys to success in the AP Calculus AB course (and on the AP Exam in May) is the ability to justify your results mathematically.

If you find that there are specific topics that you need to review in more depth, you may want to spend some time doing so. Please consider this in planning your time to work.

You are welcome to receive help on any of the problems or topics covered.

Indeed, if you have difficulty, we encourage you to work with a parent, older sibling, friend, or teacher so that you master each topic. You may consult a textbook or online resource if you wish as you work through the problems. **Be sure, however, to show all your work, and that you understand all the work you present. Note that you will have a graded classroom test on this material shortly after the start of school in September.**

We do not expect this packet to be a burden for you. However, we know that reviewing these topics will put you in a much better position to succeed in AP Calculus AB.

Best wishes for a happy rest of the summer.

AP Calculus AB Summer Problem Set

Composition of Functions

Given: $f(x) = x^2 - 4$ $g(x) = x + 2$;

1. find $f \circ g(x)$ 2. find $g \circ f(x)$ 3. find $f \circ g(3)$

Domain and Range

State the domain and range of each function:

4. $f(x) = \sqrt{x-5}$ 5. $f(x) = |x + 9| - 3$

Functions

6. In which of the following is y a function of x ?

- a. $y = 3x^2 - 9$ b. $x^2 + y^2 = 7$ c. $x^2 - y^2 = 2$
d. $3x + 2y = 5$ e. $|x| = y$

7. Determine whether the function $f(x) = -x^4 + 2x^2 - 1$ is even, odd, or neither. Justify your answer.

Use the following functions for questions 8 – 17.

$$f(x) = \frac{1}{2}x^3 - 7x^2 + 9x + 7 \quad g(x) = \frac{2}{3}x - 8$$

Find the solution set for the following:

You may use desmos or a graphing calculator to help.

8. $f(x) = 0$ 9. $f(x) = g(x)$ 10. $g(x) = 0$
11. $f(x) > -5$ 12. $g(x) \geq f(x)$ 13. $f(x) > g(x)$
14. Over what interval(s) is $f(x)$ increasing?
15. Over what interval(s) is $f(x)$ decreasing?
16. What is/are the relative maximum(s) for $f(x)$?
17. What is/are the relative minimum(s) for $f(x)$?

Use the following functions for questions 18 – 24.

$$f(x) = 3\cos(2x + \pi) \left[-\frac{\pi}{6}, \frac{7\pi}{4} \right] \quad g(x) = \frac{9}{8}x - 4$$

Find the solution set for the following:

You may use desmos or a graphing calculator to help.

18. $f(x) = g(x)$

19. $f(x) \leq g(x)$

20. $f(x) = 0$

21. $g(x) \leq f(x)$

22. $f(x) > 3$

23. $g(x) = 0$

24. Given: $f(x) = 3x^2 + 4$ and $g(x) = \frac{3x+2}{2x-1}$, find:

a) $f(2)$

b) $g(x+3)$

c) $g(f(a))$

d) $\frac{f(x+2) - f(x)}{h}$

25. Let $f(x) = \begin{cases} |x|, & x < 2 \\ x-3, & x \geq 2 \end{cases}$. Evaluate:

a. $f(-3)$

b. $f(-2)$

c. $f(0)$

d. $f(2)$

26. Find an equation of the line in point-slope form that passes through the points $(21, -12)$ and $(-14, -7)$.

Inverse Functions

27. Find the inverse of $f(x) = \sqrt[3]{1+x^3}$

Rational Functions

Name any discontinuities (holes or vertical asymptotes), horizontal or slant asymptotes for each of the following:

$$28. y = \frac{x^2 + x - 6}{x^2 + 2x - 8}$$

$$29. y = \frac{x^2 + 3x - 4}{x + 7}$$

$$30. y = \frac{x - 4}{x^2 + 9x - 22}$$

$$31. \text{Simplify the following **AND** state the Domain: } y = \frac{(x^3 + 27)(x^2 - 16)}{(x^3 + 4x^2 - 9x - 36)}$$

Expanding and Condensing Logs

$$32. \text{Write as a single logarithm: } 2\log_4 (x-3) + \log_4 (2x)$$

$$33. \text{Write the expression } \ln \left[\frac{x^4 (x+1)^3}{(x-1)^4} \right] \text{ in expanded form.}$$

Solving Exponential and Logarithmic Equations

Solve for x:

$$34. 7^{x+4} = 96$$

$$35. \log_3 17 = x$$

$$36. e^{3x} = 28$$

$$37. \text{Solve for the exact value of x without your calculator: } e^{4x+1} = 3.$$

Trig

Find the exact values of the following:

$$38. \sin 240^\circ$$

$$39. \sec \frac{-\pi}{6}$$

$$40. \tan 180^\circ$$

Prove the following identity:

$$41. \frac{\cos^2 \theta}{1 + \sin \theta} = 1 - \sin \theta$$

42. Write the following expression in algebraic form $\sin(\arccos(3x))$.

Hint: Draw a triangle in QI. Also, Arccosine is the same as \cos^{-1} . *This is inverse trig*

43. Find the exact value without your calculator: $\cos[\arctan(-\frac{3}{10})]$.

Hint: Draw a triangle in the appropriate quadrant